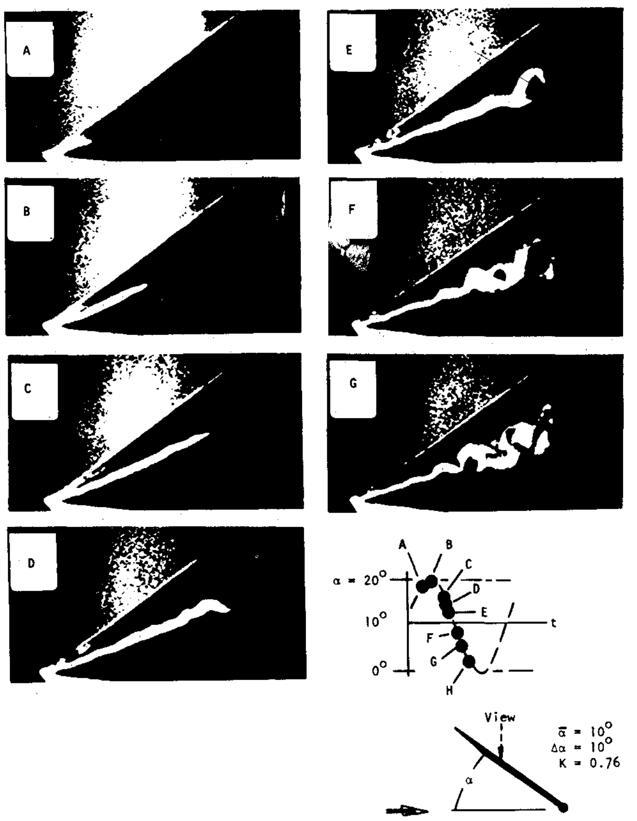

Fig. 1 Visualization of vortex breakdown on an oscillating swept wing.

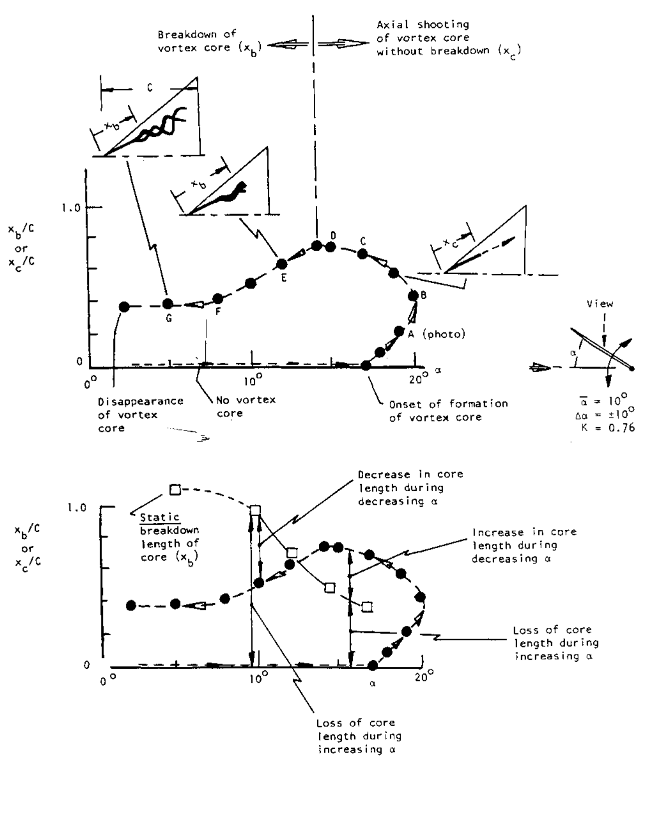

Fig. 2 Hysteresis loop corresponding to vortex breakdown on an oscillating swept wing.

core formation, as well as the location x_b of vortex core breakdown after the core has formed. The designations A, B, ... along the curve refer to the corresponding photos of Fig. 1; likewise, the schematics refer to the same series of photos. The bottom plot of Fig. 2 compares the curve of the top plot with a curve representing vortex breakdown position on the stationary wing. Increase and decrease in core length prior to breakdown, as well as complete loss of the core are indicated using the stationary wing characteristic as a reference.

Clearly, the degree to which these observations persist at other values of pitching axis location, reduced frequency, and amplitude of oscillation remains to be quantified. However, the basic features shown herein are expected to exert a significant influence on the wing loading over a substantial range.

Acknowledgment

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Extrapolation of Velocity for Inviscid Solid Boundary Conditions

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Introduction

In the computation of an inviscid flow, the solid boundary condition requires that there be no flux across the boundary, that is,

$$V \cdot n = 0 \tag{1}$$

where V is the velocity vector and n the normal vector at the solid-body surface. Equation (1) prescribes only the direction of a resultant velocity, which is sufficient for ensuring the conservation of mass and momentum. However, the magnitude of the velocity components on the body surface is very often needed for the numerical calculation for a one-sided differencing scheme or sometimes for the calculation of pressure or other flow properties, depending upon the scheme used in the calculation. An additional numerical condition is required to decompose the components of velocity on the surface.

A simple way to implement the additional numerical condition is to assume that the points near the body are close enough so that an extrapolation can be used. Nevertheless, an extrapolation is only an approximation. There are two points concerning extrapolation that must be addressed. The first is the order of extrapolation. One can employ various forms of extrapolation, for instance, a linear or higher order of extrapolation. However, except when one has information about the flow behavior near the solid body, a linear extrapolation is as good as any higher-order extrapolation. The linear extrapolation is usually applied in the computational space and not in the physical space. This simplification either relies on the assumption of small variations of the mesh spacing near the body or can be improved by inclusion of the variation of the mesh spacing. The second point is the orientation of extrapolation. It is assumed that an extrapolation from locations 2 and 3 to find a value at location 1 (see Fig. 1) will produce the same result from either grid line M or N. This is a big assumption. Its validity lies on a "good" grid such that the grid line M or N is, or is as closely as possible to be, straight and normal to the body surface. The question now is what quantity is to be linearly extrapolated.

In Ref. 1, "tangential contravariant velocity components were extrapolated" and, in Ref. 2, the tangential velocity at the body surface was obtained through linear extrapolation. The purpose of this Note is to analyze these two extrapolations, to point out their consequences, and to study the effects of an extrapolation of velocity components projected along an aribtrary direction. This will provide the information needed to implement a simpler and/or more mesh-independent extrapolation. For simplicity of discussion, only the details for a two-dimensional geometry will be presented. The arguments presented here are also valid for a three-dimensional case.

Analysis

As shown in Fig. 2, a covariant base vector, $g_i = \partial r/\partial x^i$, is tangent to its corresponding coordinate line x^i and a contravariant base vector, $g^i = \nabla x^i$, is normal to the other coordinate lines x^j, $j \neq i$. They have the relation

$$g^i \cdot g_j = \delta^i_j$$

where the indexes i and j correspond to the curvilinear coordinates x^i and x^j, which are the same as ξ and η in Fig. 2. The contravariant vector is tangent to the coordinate line only for an orthogonal mesh. Moreover, in general, the contravariant vectors are not normalized unit vectors and the so-called contravariant velocity "components"

$$V^\xi = V \cdot g^\xi = (u\xi_x + v\xi_y)$$

$$V^\eta = V \cdot g^\eta = (u\eta_x + v\eta_y)$$

are not the components of a physical velocity. Here u and v are the velocity components in the Cartesian coordinates (x,y). The "tangential contravariant velocity" adopted in Ref. 1 is interpreted as the contravariant velocity associated with the coordinates in the body-surface direction (two in the three-dimensional case); its extrapolation is expressed as

$$V_1 \cdot g_1^\xi = 2V_2 \cdot g_2^\xi - V_3 \cdot g_3^\xi$$

where the subscripts 1, 2, and 3 indicate the mesh locations at η_1, η_2, and η_3 respectively, where η_1 is the body surface. This procedure not only extrapolates the velocity components that, in general, are not tangent to the body surface (see later detailed discussion), but also extrapolates (not in a linear sense) their associated, local contravariant base vectors. This may result in a severe mesh-dependent solution.

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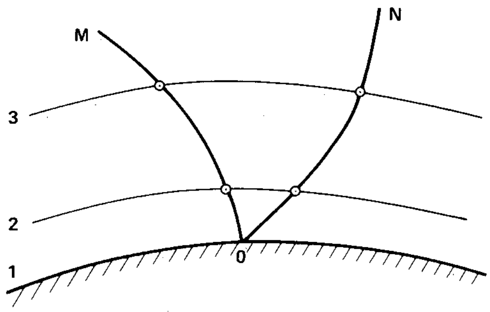

Fig. 1 Two-point linear extrapolation, $\phi_1 = 2\phi_2 - \phi_3$.

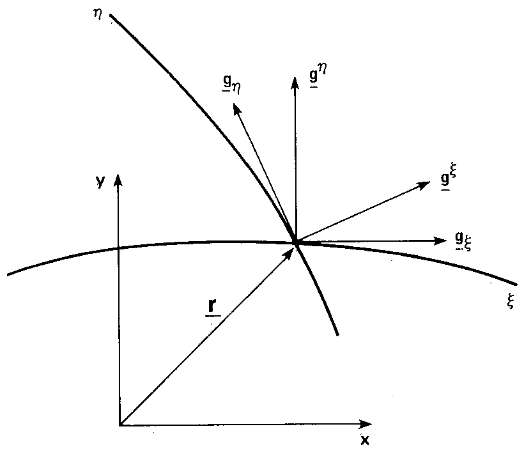

Fig. 2 Contravariant and covariant base vectors.

Reference 2 realized these deficiencies and employed the extrapolation of tangential velocity components, i.e.,

$$V_{t1} = 2V_{t2} - V_{t3} \tag{2}$$

where the tangential velocity component is

$$V_t = V \cdot g_\xi / |g_\xi|$$

However, the mesh lines η_2, and η_3 are not necessarily parallel to each other and may not be parallel to the body surface η_1. The normalization removes only the metric variations associated with magnitude, not direction. (Figure 3 is a sketch of the directions of the contravariant and covariant base vectors and the body tangent.) Therefore, the extrapolation in Eq. (2) is dependent on the direction of the mesh lines η_2 and η_3.

Now, as can be seen from Fig. 3, to avoid the dependency on the orientation of the mesh lines η_2 and η_3, one should extrapolate the velocity components in the same direction. In Eq. (2), the direction that is independent of the mesh system is the body tangent, $g_{\xi 1}/|g_{\xi 1}|$. Hence, by using the body tangent instead of local tangents, Eq. (2) is changed to

$$\frac{V_1 \cdot g_{\xi 1}}{|g_{\xi 1}|} = \frac{2V_2 \cdot g_{\xi 1}}{|g_{\xi 1}|} - \frac{V_3 \cdot g_{\xi 1}}{|g_{\xi 1}|}$$

Since only the covariant base vector $g_{\xi 1}$ at η_1 is used, its normalization is not necessary (or it can even be scaled by another artibrary constant). It becomes

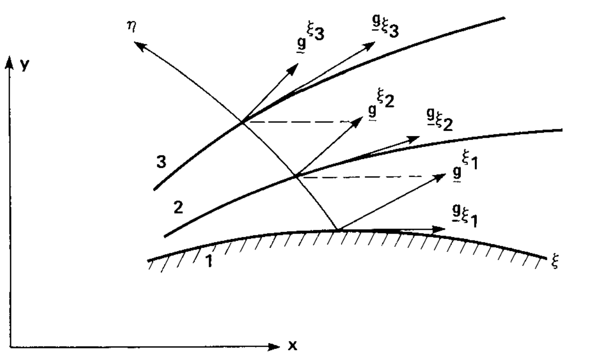

Fig. 3 Contravariant and covariant base vectors at three η locations.

$$V_1 \cdot g_{\xi 1} = (2V_2 - V_3) \cdot g_{\xi 1} \tag{3}$$

From the above discussion, one realizes that the removal of metric variation (by normalization) is needed only when a local metric is used. Note that, in a stretched rectangular grid, $g^{\xi 1}$, $g^{\xi 2}$, and $g^{\xi 3}$ are equal, and are parallel (and inverse in magnitude) to $g_{\xi 1}$. Therefore, the extrapolations used in Refs. 1 and 2 and in Eq. (3) will result in the same solution.

Next, we generalize the extrapolation and analyze the consequence of extrapolating such a velocity component, which may not be tangent to the body surface. As pointed out earlier, to be independent of the direction of mesh lines at locations 2 and 3, the only requirement is that we extrapolate velocity components in the same direction. Hence, one can substitute an arbitrary vector A for the covariant base vector $g_{\xi 1}$ in Eq. (3). Writing the decomposition procedure explicitly, the no-flux boundary condition [Eq. (1)] becomes

$$u_1 \eta_{x1} + v_1 \eta_{y1} = 0 \tag{4}$$

and the extrapolation of velocity components along a vector $A = (A_x e_i + A_y e_j)$ is

$$V_1 \cdot (A_x e_i + A_y e_j) = (2V_2 - V_3) \cdot (A_x e_i + A_y e_j)$$

or

$$A_x u_1 + A_y v_1 = A_x(2u_2 - u_3) + A_y(2v_2 - v_3) \tag{5}$$

Here e_i and e_j are the unit Cartesian base vectors. Equations (4) and (5) can be solved for u_1 and v_1, provided that A is not in the same direction as the surface normal $(\eta_{x1} e_i + \eta_{y1} e_j)$. Indeed, when A is the unit body normal, $A = (\eta_{x1} e_i + \eta_{y1} e_j)/\sqrt{\eta_{x1}^2 + \eta_{y1}^2}$, the right-hand side of Eq. (5) equals

$$R = \frac{\eta_{x1}(2u_2 - u_3) + \eta_{y1}(2v_2 - v_3)}{\sqrt{\eta_{x1}^2 + \eta_{y1}^2}}$$

and is a measure of error because of the inconsistency of the extrapolation of the normal velocity component. The employment of extrapolation as an extra numerical boundary condition indeed is based on the assumption that the error R, or strictly speaking $|R| / \| (2V_2 - V_3) \|$, is small. If $R = 0$, the tangential condition is automatically satisfied; therefore, one can extrapolate velocity components projected along any arbitrary direction (except body normal, of course) and the result will be the same. Actually, the general solution of Eqs. (4) and (5) can be expressed as

$$V_1 \cdot \frac{g_{\xi 1}}{|g_{\xi 1}|} = (2V_2 - V_3) \cdot \frac{g_{\xi 1}}{|g_{\xi 1}|} + \frac{R}{\tan\theta} \tag{6}$$

where θ is the angle between A and the body normal. The last term in Eq. (6) is an error contributed by the inconsistency error R and the arbitrary direction of vector A. If A is tangent to the body surface, then the contribution of R in the calculation V_1 is zero resulting [as in Eq. (3)] in the most appropriate velocity component to be extrapolated. In other words, if one wants to enforce a zero normal velocity, the most appropriate procedure, obviously, is to extrapolate the velocity components tangent to the body surface. Now it becomes clear that, in a severely skewed mesh, θ is small and, hence, an extrapolation of the velocity component in the direction of the contravariant base vector $g^{\xi 1}$ will result in a large error. Therefore, it is mainly the extrapolation of the tangential velocity, instead of the contravariant velocity (not the removal of metric variation as claimed in Ref. 2) that decreases the error in the extrapolation for nonorthogonal meshes.

Conclusions

In this Note, we have derived a generalization of extrapolation along an arbitrary direction [Eq. (6)]. Examination of this generalization has clearly pointed out the error associated with the arbitrary direction and has shown that the most appropriate procedure is to extrapolate the velocity components tangent to the body surface. Note that, in a typical calculation, mesh lines are in general clustered and hence are parallel or almost parallel to the body surface. Therefore, the extrapolation used in Ref. 2 will be very close to the discussed extrapolation of velocity components tangent to the body surface, except now the normalization is not necessary.

Caution should be taken that we have confined our discussions only to issues associated with simple linear extrapolation in computational space. The issues of grid line orientation and how to implement an extrapolation, as pointed out in the Introduction, could be more important. All of these indicate room for improvement and also point to the urgency and essentiality of a "good" grid near the body. As a rule of thumb, a grid system should have fine resolution, have minimal variation of mesh spacing near the body, and be nearly straight and normal to the solid body. This is essential for a computation, but is a difficult requirement, especially for a highly irregular geometry.

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Stability of Normal Shock Waves in Diffusers

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